Local Refractive Index Structure-function Parameter and its Application to Wave Propagation

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Abstract

Large-eddy simulation (LES) can provide the energy-containing, three-dimensional, time-dependent turbulent fields of the atmospheric boundary layer (ABL) that are useful in studying the diffusion of contaminants and effects of atmospheric turbulence on propagation of electro-magnetic, electro-optical, and acoustic waves. We focus on the local structure of the refractive index field and its local structure-function parameter in a convective atmospheric boundary layer. We use the local structure-function parameter in enhancing the instantaneous vertical LES profiles of refractive index fluctuations to account for turbulence fine structure. These enhanced refractive-index fields should be useful in PE calculations of wave propagation through the ABL.

1. Introduction

Large-eddy simulation is a powerful technique that provides three-dimensional, time-dependent, energy-containing fields of the atmospheric-boundary-layer turbulence. Such information has been very useful in analyzing turbulence closures (Moeng and Wyngaard, 1989) and in understanding the instantaneous local structure of the atmospheric boundary layer (Schmidt and Schumann, 1989; Khanna and Brasseur, 1997). However, LES has not been fully explored for studying the effects of atmospheric turbulence on wave propagation, in part because of the insufficient spatial resolution of LES data.

The propagation of electro-magnetic waves, for example, is influenced by local fluctuations in the refractive index n. A statistical measure of these local fluctuations in the inertial subrange of spatial scales is the local refractive index structure-function parameter $\tilde{C}_{N^2}$ introduced by Peltier and Wyngaard (1995). It is defined as

$$\tilde{C}_{N^2} = 1.6\tilde{\epsilon}^{-1/3}\tilde{\chi}_n, \tag{1}$$

where $\tilde{\epsilon}$ is the local volume-averaged dissipation rate of turbulent kinetic energy, and $\tilde{\chi}_n$ is the local volume-averaged destruction rate of refractive-index variance. $\tilde{C}_{N^2}$ is a three-dimensional, time-dependent, fluctuating quantity as opposed to the ensemble-mean structure-function parameter C_{N^2} defined through the ensemeble-mean quantities ϵ and χ_n and traditionally used in wave-propagation studies (Tatarskii, 1971). Further discussion on local volume-averaging and the relevance of $\tilde{C}_{N^2}$ to wave propagation is provided in section 2.

For propagation applications, for example PE calculations (Gilbert et al., 1996), one often needs refractive index fields that are vertically well resolved (on roughly one-meter scales), since the refractive-index gradients are largest at these small length scales. LES typically provides fields that are spatially resolved to 20 – 50 m. Typical LES profiles, "therefore, can lack the fine structure significant for propagation applications. We will use the local refractive index structure-function parameter in adding fine structure to the instantaneous LES profiles.

We present the local structure of $\tilde{C}_{N^2}$ in a convective boundary layer ($-z_i/L \approx 450$) simulated over a 5 km x 5 km x 2 km domain using Moeng (1984) LES code. We also show the LES profiles of refractive index fluctuations at arbitrary X-Y locations in the horizontally homogeneous boundary layer and present the enhanced versions of these profiles.

2. Local Structure-function Parameters

In large-eddy simulation as well as remote sensing one simulates or measures locally averaged quantities. For example, in LES the resolvable-scale velocity is defined as

$$u_i^r(\vec{X}) = \int_{-\infty}^{+\infty} u_i(\vec{X}-\vec{Y})G(\vec{Y})\, d\vec{Y}, \tag{2}$$

where u_i is the velocity, a random variable, and G is a spatial filter function. The simplest filter function is that of local grid-volume averaging:

$$\begin{aligned} G(\vec{X}) &= 1; \qquad & |X| &\leq r \\ G(\vec{X}) &= 0; \qquad & |X| &> r. \end{aligned} \tag{3}$$

In typical LES applications, r is of the order of 20-50 m.

In remote sensing the scattered intensity from a small volume in the boundary layer (Tatarskii, 1971) is given by

$$E_s(\vec{k},\vec{r},t) \propto \int_V e^{i\vec{k}\cdot\vec{x}} n(\vec{x})\, d\vec{x} \int_V e^{\,i\vec{k}\cdot\vec{y}} n(\vec{y})\, d\vec{y}, \tag{4}$$

where k depends on the frequency of propagating wave and the orientation of the source and the receiver with respect to the scattering volume as shown in figure 1. If the characteristic length scale of the scattering volume is large compared to the propagating wave length, (4) can be approximated as

$$E_s(\vec{k},\vec{r},t) \propto \hat{n}_v(\vec{k},\vec{r},t)\, \hat{n}_v^*(\vec{k},\vec{r},t), \tag{5}$$

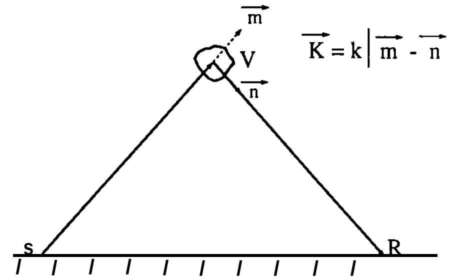

Figure **1** A schematic of scattering geometry. S is the source; R the receiver; V the scattering volume; $\vec{m}$ and $\vec{n}$ the unit vectors; and k the wavenumber.

where $\hat{n}_v$ is the Fourier coefficient of n (Batchelor, 1953). Locally averaged E_s, averaged over different directions of k or time smaller than the subgrid-eddy characteristic time scale (Wilson et al., 1996), is then given by

$$E_s(\vec{k}, \vec{r}, t) \propto \overline{\Phi}_n(\vec{k})|_{\tilde{C}_{N^2}} = \tilde{C}_{N^2}(\vec{r}) k^{-11/3}, \tag{6}$$

where ~.(~) $|_{\tilde{C}_{N^2}}$ is the conditionally averaged spectral density function defined by Kolmogorov (1962) and $\tilde{C}_{N^2}$ is the local structure-function parameter as defined by Peltier and W yngaard (1996). The scattered intensity, is therefore proportional to the local structure-function parameter.

3. Preliminary Results

To study the local structure of $\tilde{C}_{N^2}$ in a convective boundary layer over ocean, we carried out a 128^3 simulation over a 5 km x 5 km x 2 km domain with mild geostrophic wind speed (1 m/s) and strong surface temperature and moisture fluxes (0.24 m/s K and 0.08 m/s (g of vapor/kg of air), respectively). The boundary layer depth was roughly 700 m and the Monin-Obukhov length scale L was 1.5 m. Due to the entrainment of dry air aloft, the moisture flux at the capping inversion was nearly same as its surface flux; the temperature flux at the capping inversion was roughly 10 % of its surface value.

Figure 2 shows the isosurfaces of $\tilde{C}_{N^2}$ along a vertical plane covering the entire boundary layer and along a horizontal plane near the ground. The values are particularly large in the interracial layer, thereby highlighting its instantaneous local structure. The near-ground structure forms a cellular pattern characteristic of Raleigh-Bénard convection.

Figure 3 shows the isosurfaces of the upward vertical velocity fluctuations within the mixed layer and the isocontours of $\tilde{C}_{N^2}$ along a horizontal plane (semi-opaque) within the interracial layer. The figure indicates that the C_{N^2} values within the interracial layer are

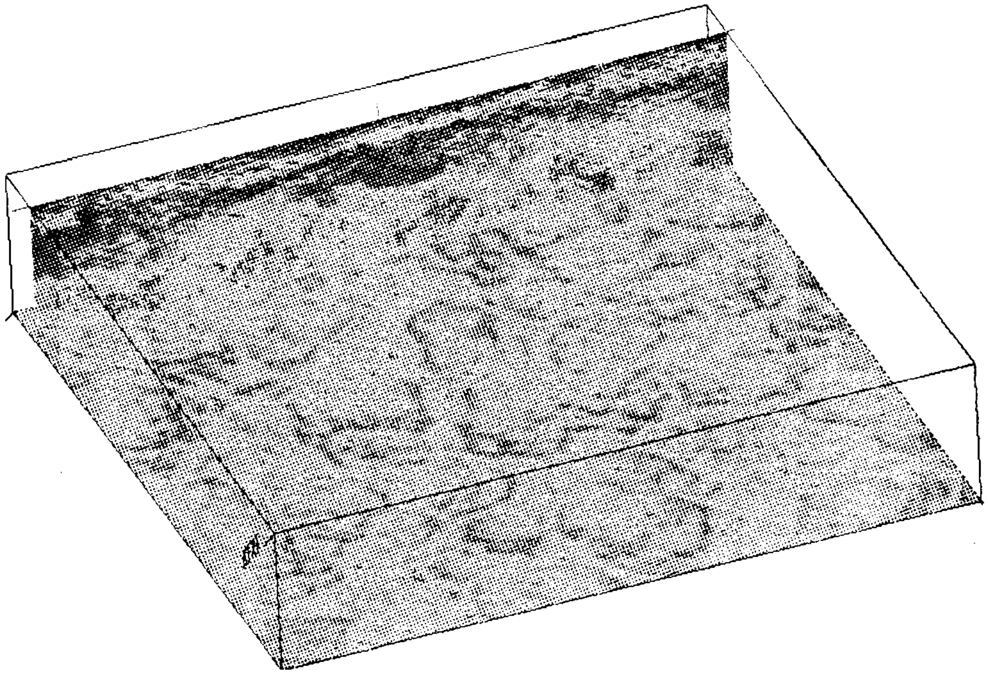

Figure 2 Isocontours of the local refractive index structure-function parameter along a horizontal and a vertical plane in a convective boundary layer. Red represents the most intense region and blue the least intense.

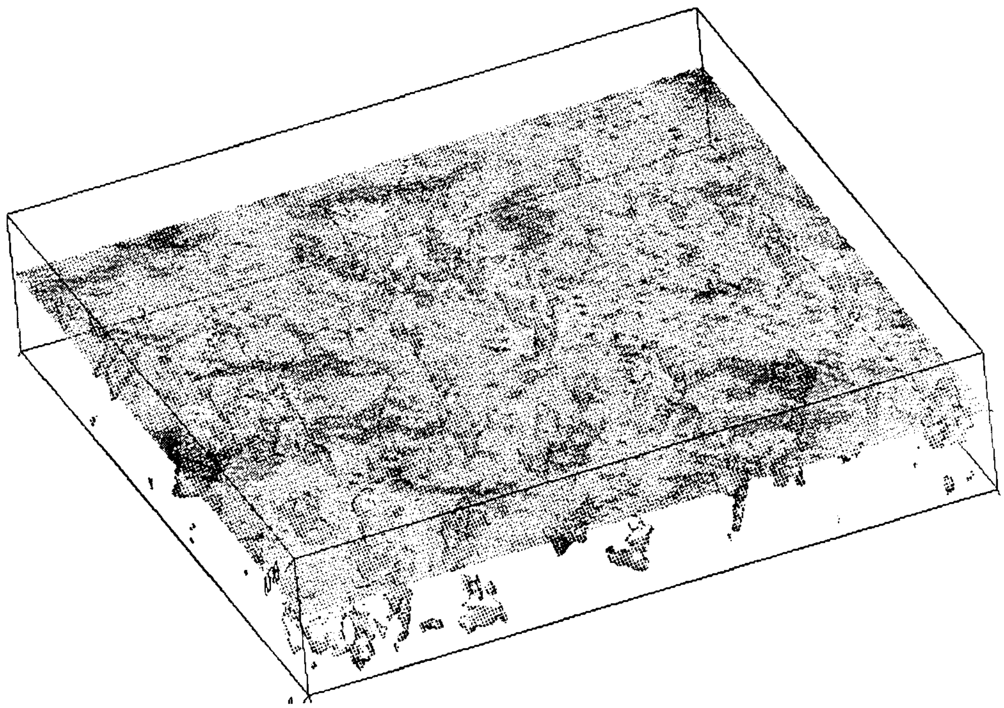

Figure **3** Isosurfaces of the positive vertical velocity (yellow) within the mixed layer and -- isocontours of the local refractive index structure-function parameter along a horizontal plane (semi-opaque) within the capping inversion. For the isocontours, red represents the most intense region and blue the least intense.

particularly large around the thermals as they penetrate the capping inversion.

We use these local structure-function parameters in adding fine structure to the instantaneous refractive-index profiles from LES in order to make them useful in applications to signal propagation. In our technique, we assume that the turbulence scales smaller than the LES grid size are locally isotropic and hence exhibit the classical inertial subrange $\kappa^{-5/3}$ scaling. To account for intermittence effects, as suggested by Kolmogorov (1962), we use the local structure-function parameters to set the inertial-range amplitude. We add Gaussian variability within these ensemble-mean arguments to account for local fluctuations. A manuscript discussing our technique for enhancing fine structure is under preparation.

Figure 4 shows the instantaneous vertical profiles of refractive index as obtained directly from LES and after we enhanced the fine structure. The figure shows the fine-scale enhancement depends on the large-scale features predicted by LES; this is a consequence of using the local structure-function parameter to determine the amplitude of the added fine structure. We believe the enhanced refractive-index profiles from LES can form a powerful data base for studying the effects of turbulence on EM and acoustic wave propagation.

4. Summary

We focus on the potential of large-eddy simulation for providing instantaneous three-dimensional data sets for analyzing the affects of atmospheric turbulence on EM, EO, or acoustic wave propagation. We discuss the relevance of local structure function parameters to propagation applications, and analyze their instantaneous local behavior in some detail. We show an application of the local structure-function parameters in enhancing the instantaneous LES profiles to account for turbulence fine structure. Further studies using the enhanced profiles are currently underway.

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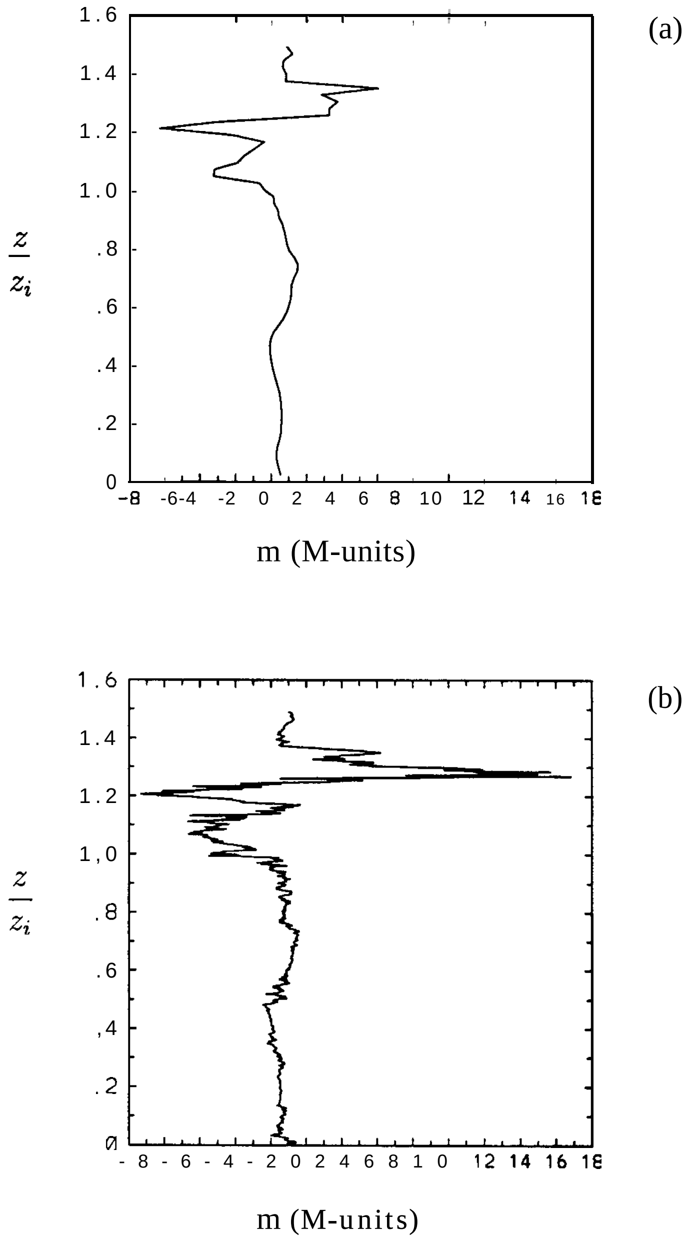

Figure 41nstantaneous profiles of the modified refractivity fluctuations, $(n-1)\times 10^6$, in a convective boundary layer: (a) resolvable profile from LES, and (b) enhanced profile including subgrid-scale contribution.

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